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# Research on two different mathematical theories on control

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**Abstract**

With a brand-new theory, this paper not only provides the differences of attributes in concept, formula expression and function type between fuzzy rough sets and probability statistics, but also introduces their differences in algorithms on target control for better solving the control problem. Some new definitions and theorems concerning fuzzy rough sets and probability statistics are given, but this paper mainly makes a comparison of two control algorithms for the target tracking. The simulation results show that the comprehensive performance of the fuzzy rough sets algorithm is better than that of the probability statistics algorithm, but its control effect is not as good as that of the latter on multisensor target control. Finally, some problems concerning the combination of fuzzy rough sets and the probability statistics phenomenon to be solved and development trends are discussed. By these investigations, we can choose the optimal control algorithms for accomplishing better target control.


*Keywords:* Fuzzy rough sets; Probability statistics; Target tracking; Decision control; Decision rule; Decision degree

## 1. Introduction

In target control, there are various algorithms, but to accomplish tasks better, what algorithm we choose is a significant problem. In the past, the probability statistics (PS) method [11,22–25] was usually used for target control. Along with the development of information and industry technology, at present, people are becoming more and more interested in fuzzy control [1,3–10,12,13,15,17–19,21,26] and rough sets for target control [2,20,27,28]; again, by the combination of fuzzy sets (FS) theory and rough sets (RS) theory, i.e., fuzzy rough sets (FRS) theory [14,16,29], we can obtain a new control algorithm, which here is called the fuzzy rough (FR) control algorithm. However, what differences are there between the new FR algorithm and the PS algorithm on target control? This needs to be studied.

This paper studies the above-mentioned problem, i.e., gives the comparison of the FR algorithm and the PS algorithm, and discusses their advantages and disadvantages using simulation. Briefly, this paper discusses mainly the differences of FR and PS algorithms on target control so that their essential difference can be understood further. What the FR algorithm on target control carries out is first selecting several optimum radars according to the maximum-membership principle, and then finishing controlling the target through the fusion of information based on the genetic

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algorithm [1]. But the PS algorithm utilizes the characteristics of mean and covariance in order to finish the target control through accurate numerical computations. At the same time, the simulations are given for the two different control algorithms. From the simulation results known, the new FR algorithm appears an optimal choice for target control when some probabilities of conditional items and statistical distributions are unknown or the requirement of precision of control is not too high, because it has a lot of advantages compared with the probabilistic algorithm, such as having faster processing speed, lower memory capacity and communications traffic; in addition, some probabilities of conditional items and statistical distributions are usually obtained with difficulty.

In this paper, these researches on the algorithms for target control not only are of important theoretical value to navigation, positioning and target tracking, but also promote applications of navigation systems and target control. That is to say, the theories developed in this paper can in turn form the theoretical basis for further applications of navigation systems and target control.

The rest of this paper is organized as follows: Some nomenclature and definitions are given in Section 2. The FR decision control algorithm is introduced in Section 3. The PS control algorithm is described in Section 4. The simulation and analysis of simulation results for the two algorithms on target control are discussed in Section 5. Finally, some conclusions and directions for future research are given in Section 6.

## 2. Definitions

### 2.1. Nomenclature

$A, B, C, A^i$ $(i = 1, 2, \ldots, n)$ — Fuzzy rough sets.
$A^c$ — A complementary set of set $A$.
$\overset{\approx}{A}$ — Upper approximation in definition of rough set [2,14].
$\underset{\approx}{A}$ — Lower approximation in definition of rough set.
$C_a$ — A condition attribute set.
$D_a$ — A decision attribute set.
$E$ — Mathematical expectation.
$\overline{e}$ — Mean value of the absolute value of tracking error.
$F$ — State transition matrix.
FR — Fuzzy rough.
FRS — Fuzzy rough sets.
FS — Fuzzy sets.
RS — Rough sets.
$G$ — Process noise transition matrix.
$H$ — Measurement matrix.
$L$ — A set of all real numbers from 0 to 1, i.e., $L = [0, 1]$.
MM — Multiple-target model.
$N$ — FR norm.
$N(A, B)$ — FR norm of FRS $A$ and $B$.
$P$ — Covariance matrix of error for state of target model.
Pr — Probability.
PS — Probability statistics.
$r$ — Decision rule.
$R$ — Equivalence relation in universal set $U$.
$R^n$ — An $n$-dimensional vector with real number value.
$S$ — Covariance of measure error.
$U$ — Universal set, i.e., a universe of discourse.
$\{U, R\}$ — Approximate space, i.e., a universe of discourse with an equivalence relation $R$.
$U/C_a$ — All equivalence classes of $C_a$.
$U/D_a$ — All equivalence classes of $D_a$.
$\nu$ — Measurement noise.

$V$ — Measurement noise covariance matrix.
$w$ — Process noise.
$W$ — Process noise covariance matrix.
$x$ — State vector.
$\overset{\Lambda}{x}$ — Estimate value of state for target.
$\overline{x}$ — Mean value of state estimate.
$\widetilde{x}$ — Error for state estimate.
$z$ — Measurement vector.
$\emptyset$ — Empty set.
$\Gamma$ — Event domain.
$\mu$ — Probability of model.
$\pi$ — State transition probability of target model.
$\Psi$ — Control matrix.
$\triangleq$ — An operator for defining the equality.
$\Omega$ — A closed domain.
$\rho$ — Decision degree of rule.

### 2.2. Preliminary FRS and PS

The notion of RS was first introduced by Pawlak [27] and subsequently the algebraic approach to RS was given by Iwinski [28]. The concept of FS was first introduced by Zadeh [21]. Pawlak provided the comparison between FS and RS and showed the difference between the two [29]. The definition of FRS is given by [2,14]. But to allow easy discussion of the following problem, here we give a new equivalent definition of [2,14] for FRS.

The FS is defined as follows:

**Definition 2.1.** Let $U$ be a non-empty universal set and assume $L = [0, 1]$. A fuzzy set $A$ in $U$ is a mapping $A : U \to L$ (or $u \longmapsto A(u)$).

For every $u \in U$, $A(u) \in L$ is the degree of membership of an element $u \in U$ in the fuzzy set $A$.

**Definition 2.2.** Assume $L = [0, 1]$. Let $\{U, R\}$ be a given approximate space. A set $A$ is called a fuzzy rough set in $\{U, R\}$ or in $U$ if it satisfies the following conditions:

① $A$ is a rough set on $\{U, R\}$ which consists of $\underset{\approx}{A}$ and $\overset{\approx}{A}$, and $A$ is represented as $A = \langle \underset{\approx}{A}, \overset{\approx}{A} \rangle$ with $\underset{\approx}{A} \subseteq \overset{\approx}{A}$.

② $A$is also a fuzzy set in $U$, i.e., $A$ is characterized by a pair of mappings $\underset{\approx}{A} : U \to L$ and $\overset{\approx}{A} : U \to L$.

Then the set $A$ is called a fuzzy rough set in $U$.

According to $\underset{\approx}{A} \subseteq A \subseteq \overset{\approx}{A}$, this implies $\underset{\approx}{A}(x) \leqslant A(x) \leqslant \overset{\approx}{A}(x)$ for all $x \in U$.

All FRS in $U$ are represented as $FR(U)$, i.e., $FR(U) = \{A | A = \langle \underset{\approx}{A}, \overset{\approx}{A} \rangle, \underset{\approx}{A} : U \to L; \overset{\approx}{A} : U \to L\}$.

Since FS do not satisfy the complementary law [21], FRS do not satisfy the complementary law either, i.e., each has no complement in $(FR(U), \bigcup, \bigcap, c)$. $A \bigcap A^c \neq \emptyset$ shows that $A$ and $A^c$overlap, but we have $A(u) \bigwedge A^c(u) \leqslant \frac{1}{2}$ for any $A \in FR(U)$, where $\forall u \in U$, $A^c$ is a complementary set for $A$. Similarly, we have $A \bigcup A^c \neq U$, but also $A(u) \bigvee A^c(u) \geqslant \frac{1}{2}$ for any $A \in FR(U)$, where $\forall u \in U$.

From the above discussion, we have

$$\frac{1}{2} \leqslant \left(A \bigcup A^c\right)(u) \leqslant 1, \qquad 0 \leqslant \left(A \bigcap A^c\right)(u) \leqslant \frac{1}{2}.$$

Since the complementary operation of FRS is unsatisfied with the complementary law, it can more objectively describe and handle ambiguous things in many real-world problems than ordinary sets.

Probability statistics is linked to the event, so we next introduce the probability event first.

**Definition 2.3** (*[11]*). An event is called a probability event if it satisfies happening or not happening in the experimental results.

From the description of the probability definition, the probability of a probability event $A$ is $\Pr(A) \in [0, 1]$.

Now, the exact mathematical definition of probability is given as follows:

**Definition 2.4.** Let $(\Omega, \Gamma, \Pr)$ be a given probability space, where $\Omega$ is a sample set, $\Gamma$ is an event domain and Pr is a probability measure. Let $[0, 1]$ be a given real number set. There is a mapping from $\Gamma$ to $[0, 1]$:

$$\Pr : \Gamma \to [0, 1]$$
$$\Pr : A \to \Pr(A), \quad \forall A \in \Gamma.$$

Then Pr is called a probability in $\Gamma$.

For $\forall A \in \Gamma$, $\Pr(A)$ is called a probability of event $A$, i.e., for any given event $A$ in $\Gamma$, there is a corresponding probability $\Pr(A)$. Therefore, the probability defined in event domain $\Gamma$ is a set function $\Pr(\bullet)$ of real number value.

The set function $\Pr(\bullet)$ satisfies the following conditions [11]:

(1) Non-negative feature: for each event $A$, $\Pr(A) \geqslant 0$.
(2) Norm attribute: $\Pr(\Omega) = 1$.
(3) Addition feature: if $A_1, A_2, \ldots, A_n, \ldots$ are the sets of any two incompatible events,

$$\Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Pr(A_i).$$

For $A \in \Gamma$, $\Pr(A) + \Pr(A^c) = 1$, where $A^c$ is an event opposite to event $A$.

For the above-mentioned definitions, we have conclusions: The fuzzy rough theory and the probability theory are two different theories. The probability event is unknown and uncertain before testing, while the testing results are known after testing. However, the fuzzy rough case itself is ambiguous and has no definite boundary.

Previously, the concepts of FR and probabilistic phenomena were believed identical, i.e., the probabilistic phenomena were believed fuzzy or rough. However, the FRS and the probability event are different in nature and their differences are inherent. The probability event mostly divides the probability space for non-deterministic information on the basis of probability and expectation mean theory and Bayes' rule. In other words, we have to estimate the degree of certainty when the evidence is actually incomplete or subjective; thus the correctness of the evidence that is obtained is more or less artificial, by the partitioning of the probability space. This is probably a main weakness of the probabilistic formulation. But its main strengths include that it is rigorous, systematic, and particularly suitable for random sequence processing. As a result, in the context of multiple-target model (MM) estimation [22,23], it may potentially offer a framework for handling better a mismatch with the true model. However, the FR approach offers a variety of alternatives with distinctive flexibilities, which are valuable for handling many problems of real life based on fuzzy functions and degree of membership. Like for the FR approach, however, its degrees of membership are very artificial and not more accurate than the probability value, that is to say, the degrees of membership of FR are usually obtained subjectively from some experience and experiments, while the probability value is usually obtained by accurate numerical calculation based on the probability formula and Bayes' formulas, etc.; although the degrees of membership of FR can be obtained from the FR membership function, the FR membership functions are given from subjective experience of an expert system. At the same time, this does not provide a way of fusing old knowledge and new information as Bayes' rule does in the probabilistic setting. When the integration of the old evidence and new data is necessary in the sequence process, the FR approach is not as good as the probability approach for target control under conditional control with perfect knowledge of the target model. As a result, as a side-effect of its flexibility, the FR approach is more open to misuse than the probability statistics (PS) approach. The inferior performance of the FR approach as compared with the standard PS approach is indicated by the simulation results. A main weakness of the FR approach arises from the lack of solid, systematic weight update, while the PS approach has a built-in solid mechanism for sequential update of the weights thanks to Bayes' rule. The FRS relies on heuristic principles and expert systems for weight update, while the "mixture of experts" of [24,25] had recourse to an optimization searching tree algorithm with the help of Bayes' rule.

## 3. FR control algorithm

### 3.1. Basis knowledge for FR control

According to the definition of the degree of approach in [17], the definition of the FR norm and a concrete FR norm are introduced as follows.

**Definition 3.1.** Let $\{U, R\}$ be a given approximate space. Assume $A, B, C \in FR(U)$ and let $L = [0, 1]$. $N(A, B)$ is called a FR norm of FRS $A$ and $B$ if the mapping $N : FR(U) \times FR(U) \to L$ satisfies the following conditions:

① $N(A, B) = N(B, A) \Longleftrightarrow N(\underset{\approx}{A}, \underset{\approx}{B}) = N(\underset{\approx}{B}, \underset{\approx}{A})$ and $N(\overset{\approx}{A}, \overset{\approx}{B}) = N(\overset{\approx}{B}, \overset{\approx}{A})$;

② $0 \leqslant N(A, B) \leqslant 1 \Longleftrightarrow 0 \leqslant N(\underset{\approx}{A}, \underset{\approx}{B}) \leqslant 1$ and $0 \leqslant N(\overset{\approx}{A}, \overset{\approx}{B}) \leqslant 1$,

$N(U, \emptyset) = 0 \Longleftrightarrow N(\underset{\approx}{U}, \underset{\approx}{\emptyset}) = 0$ and $N(\overset{\approx}{U}, \overset{\approx}{\emptyset}) = 0$, where $U$ is a universal set, $\emptyset$ is an empty set;

③ if $A \subseteq B \subseteq C$, then $N(A, C) \leqslant N(A, B) \bigwedge N(B, C) \Longleftrightarrow N(\underset{\approx}{A}, \underset{\approx}{C}) \leqslant N(\underset{\approx}{A}, \underset{\approx}{B}) \bigwedge N(\underset{\approx}{B}, \underset{\approx}{C})$ and $N(\overset{\approx}{A}, \overset{\approx}{C}) \leqslant N(\overset{\approx}{A}, \overset{\approx}{B}) \bigwedge N(\overset{\approx}{B}, \overset{\approx}{C})$.

Here $N$ is called a FR norm function on $FR(U)$. The approximate space $\{U, R\}$ is called a FR normed space.

According to the above definition, here we will give a type of FR norm as follows:

**Theorem 3.1.** *If $U = \{u_1, u_2, \ldots, u_n\}$, then*

$$N(A, B) \triangleq 1 - \frac{1}{n} \sum_{i=1}^{n} |A(u_i) - B(u_i)| \tag{1}$$

*is a FR norm of FRS A and B, where $A = \langle \underset{\approx}{A}, \overset{\approx}{A} \rangle$, $B = \langle \underset{\approx}{B}, \overset{\approx}{B} \rangle \in FR(U)$, $u_i \in U$, $A(u_i) = \langle \underset{\approx}{A}(u_i), \overset{\approx}{A}(u_i) \rangle$, $B(u_i) = \langle \underset{\approx}{B}(u_i), \overset{\approx}{B}(u_i) \rangle$.*

Define $A(u_i) - B(u_i) = \langle \underset{\approx}{A}(u_i) - \underset{\approx}{B}(u_i), \overset{\approx}{A}(u_i) - \overset{\approx}{B}(u_i) \rangle$.

In the real number domain, when $U$ is a closed domain $\Omega$, i.e., $U = \Omega$, then

$$N(A, B) \triangleq 1 - \frac{1}{|\Omega|} \int_{\Omega} |A(u) - B(u)| \mathrm{d}u \tag{2}$$

is a FR norm of FRS $A$ and $B$, where $|\Omega|$ is a measure of $\Omega$, which is a length, area or volume.

**Proof.** Theorem 3.1 is verified correct by computer simulation and calculation. However, its correctness is also obtained by a mathematical method. Then, the equality (1) is proved as follows:

①

$$N(\underset{\approx}{A}, \underset{\approx}{B}) = 1 - \frac{1}{n} \sum_{i=1}^{n} |\underset{\approx}{A}(u_i) - \underset{\approx}{B}(u_i)| = 1 - \frac{1}{n} \sum_{i=1}^{n} |\underset{\approx}{B}(u_i) - \underset{\approx}{A}(u_i)| = N(\underset{\approx}{B}, \underset{\approx}{A})$$

and

$$N(\overset{\approx}{A}, \overset{\approx}{B}) = N(\overset{\approx}{B}, \overset{\approx}{A}),$$

so $N(A, B) = N(B, A)$.

② Since $0 \leqslant \underset{\approx}{A}(u_i), \overset{\approx}{A}(u_i), \underset{\approx}{B}(u_i), \overset{\approx}{B}(u_i) \leqslant 1$, there are $0 \leqslant N(\underset{\approx}{A}, \underset{\approx}{B}) \leqslant 1$ and $0 \leqslant N(\overset{\approx}{A}, \overset{\approx}{B}) \leqslant 1$, so $0 \leqslant N(A, B) \leqslant 1$.

In particular, $N(\underset{\approx}{A}, \underset{\approx}{A}) = 1 - \frac{1}{n}\sum_{i=1}^{n} |\underset{\approx}{A}(u_i) - \underset{\approx}{A}(u_i)| = 1$, $N(\overset{\approx}{A}, \overset{\approx}{A}) = 1$, so $N(A, A) = 1$. Similarly, $N(\underset{\approx}{U}, \underset{\approx}{\emptyset}) = 1 - \frac{1}{n}\sum_{i=1}^{n} |\underset{\approx}{U}(u_i) - \underset{\approx}{\emptyset}(u_i)| = 1 - 1 = 0$, $N(\overset{\approx}{U}, \overset{\approx}{\emptyset}) = 0$, so $N(U, \emptyset) = 0$.

③ If $A \subseteq B \subseteq C$, then $|A(u_i) - C(u_i)| \geqslant |A(u_i) - B(u_i)|$ and $|A(u_i) - C(u_i)| \geqslant |B(u_i) - C(u_i)|$, so

$$1 - \frac{1}{n}\sum_{i=1}^{n} |A(u_i) - C(u_i)| \leqslant 1 - \frac{1}{n}\sum_{i=1}^{n} |A(u_i) - B(u_i)|$$

and

$$1 - \frac{1}{n}\sum_{i=1}^{n} |A(u_i) - C(u_i)| \leqslant 1 - \frac{1}{n}\sum_{i=1}^{n} |B(u_i) - C(u_i)|.$$

Therefore, we have $N(A, C) \leqslant N(A, B) \bigwedge N(B, C)$.

So, the equality (1) is a FR norm.

Similarly, the equality (2) can be proved. At the same time, the norm that is defined by Theorem 3.1 is called a FR 1-norm. □

### 3.2. FR control principle

Here, two methods for FR control are given. The direct method is a maximum-membership principle that is applied mainly in control of an individual. The indirect method is based on a selected-approach principle that is applied generally in control of a group model.

(1) Maximum-membership principle

**Definition 3.2.** Assume $A^i \in FR(U)$ $(i = 1, 2, \ldots, n)$. For $u_0 \in U$, if there are an $i_0$ and a $j_0$ making $\underset{\approx}{A}^{i_0}(u_0) = \max\{\underset{\approx}{A}^1(u_0), \underset{\approx}{A}^2(u_0), \ldots, \underset{\approx}{A}^n(u_0)\}$, and $\overset{\approx j_0}{A}(u_0) = \max\{\overset{\approx 1}{A}(u_0), \overset{\approx 2}{A}(u_0), \ldots, \overset{\approx n}{A}(u_0)\}$, then $u_0$ is believed to be subordinate to $\underset{\approx}{A}^{i_0}$ and $\overset{\approx j_0}{A}$. This principle is called a maximum-membership principle.

Again, according to the testing requirement and the trial and error, $A^{i_0}$ and $A^{j_0}$ are determined.

(2) Selected-approach principle

**Definition 3.3.** Let $A^i, B \in FR(U)$ $(i = 1, 2, \ldots, n)$. If there is an $i_0$ letting $N(\underset{\approx}{A}^{i_0}, \underset{\approx}{B}) = \max\{N(\underset{\approx}{A}^1, \underset{\approx}{B}), N(\underset{\approx}{A}^2, \underset{\approx}{B}), \ldots, N(\underset{\approx}{A}^n, \underset{\approx}{B})\}$ be true, then $\underset{\approx}{B}$ is believed to be most near $\underset{\approx}{A}^{i_0}$, i.e., $\underset{\approx}{B}$ and $\underset{\approx}{A}^{i_0}$ are believed to be congeneric. Similarly, if there is a $j_0$ letting $N(\overset{\approx j_0}{A}, \overset{\approx}{B}) = \max\{N(\overset{\approx 1}{A}, \overset{\approx}{B}), N(\overset{\approx 2}{A}, \overset{\approx}{B}), \ldots, N(\overset{\approx n}{A}, \overset{\approx}{B})\}$ be true, then $\overset{\approx}{B}$ and $\overset{\approx j_0}{A}$ are believed to be congeneric. This principle is called a selected-approach principle.

Similarly, according to the testing requirement or the trial and error, $A^{i_0}$ and $A^{j_0}$ are determined.

### 3.3. FR decision rule

Here, we give the decision rule on target control.

**Definition 3.4.** Assume $A = C_a \bigcup D_a$, $C_a \bigcap D_a = \emptyset$, where $C_a$ is a condition attribute set, $D_a$ is a decision attribute set. Let $X_i$ and $Y_j$ be equivalence classes of $U/C_a$ and $U/D_a$ respectively, where $U/C_a$ and $U/D_a$ denote all equivalence classes of $C_a$ and $D_a$ respectively.

The decision rule is defined by: $r_{ij} : X_i \to Y_j$, $X_i \bigcap Y_j \neq \emptyset$.

The decision degree of the rule is defined by: $\rho(X_i, Y_j) = |Y_j \bigcap X_i|/|X_i|$, and then $0 < \rho(X_i, Y_j) \leqslant 1$.

Table 1
Control decisions of some radar types

| Universal set (Target type) | Condition attribute | | | Decision attribute (Degree of control) |
|---|---|---|---|---|
| | Position | Speed | Acceleration | |
| 1 | 0.5 | 0.3 | 0.2 | Very good |
| 2 | 0.1 | 0.3 | 0.6 | Not good |
| 3 | 0.1 | 0.5 | 0.4 | Better |
| 4 | 0.3 | 0.4 | 0.3 | Better |

### 3.4. Example of FR decision control

In this section, the implementation procedure of the control for the target is illustrated by a concrete example.

**Proposition 3.1.** *Here, the control on targets is discussed with three types of radar tracking of four targets as an example. It is shown in Table 1. Here, the universal set $U$ is the types of target $1, 2, 3, 4$, i.e., $U = \{1, 2, 3, 4\}$. The condition attribute set $C_a = \{$position, speed, acceleration$\}$, the decision attributes set $D_a = \{$very good, better, not so good, not good$\}$, for controlling the target; then, what is the decision rule?*

*Solution*. For target 1, we carry out a single-factor evaluation with a number of fine radars. At a certain time, considering only the position of the target, 60% of the radars are very good for controlling targets, 20% of the radars are better for controlling targets, 10% of the radars are not so good for controlling targets and 10% of the radars control targets poorly. Therefore, we can draw a conclusion: The position $\longmapsto (0.6, 0.2, 0.1, 0.1)$.

Likewise, assume the speed $\longmapsto (0.1, 0.4, 0.3, 0.2)$ and the acceleration $\longmapsto (0.1, 0.2, 0.3, 0.4)$.

Accordingly, the control matrix is

$$\Psi = \begin{pmatrix} 0.6 & 0.2 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.3 & 0.4 \end{pmatrix}.$$

Because of various factors such as quality, the different types of radar offer different weights for three factors of targets. Let some radar types provide the weight $a_1 = (0.5, 0.3, 0.2)$ for target 1. According to these radar types for control of the target, the generalization of control can be obtained: $b_1 = a_1 \circ \Psi = (0.35, 0.26, 0.20, 0.19)$, where $\circ$ is a composition operator based on the addition and product operators for real numbers. The control $b_1$ shows that the degree of 'very good' is 35%, for controlling target 1, the degree of 'better' is 26%, the degree of 'not so good' is 20%, and the degree of 'not good' is 19%. According to the maximum-membership principle, the conclusion obtained is that the control vector $b_1$ is 'very good', for controlling target 1.

Likewise, for the other three targets, let the control matrix also be $\Psi$. The weights that these radar types provided for the three targets are respectively

$$a_2 = (0.1, 0.3, 0.6), \qquad a_3 = (0.1, 0.5, 0.4), \qquad a_4 = (0.3, 0.4, 0.3).$$

And so the corresponding synthesized control is

$$b_2 = a_2 \circ \Psi = (0.15, 0.26, 0.28, 0.31)$$
$$b_3 = a_3 \circ \Psi = (0.15, 0.30, 0.28, 0.27)$$
$$b_4 = a_4 \circ \Psi = (0.25, 0.28, 0.24, 0.23).$$

The conclusions obtained are that the control vectors $b_2$, $b_3$ and $b_4$ are 'not good', 'better' and 'better', for controlling targets 2, 3 and 4 respectively. The results are shown in Table 1.

Now, we verify whether the control matrix $\Psi$ is an optimal control matrix for the targets 2, 3 and 4. Here we only give whether the control matrix $\Psi$ is optimal for the 'position' attribute of target 2.

For target 2, with the low-level radars, middle-level radars and high-level radars for tracking it, its position-evaluated vectors are obtained as follows, respectively:

$$\beta_1 = (0.2, 0.3, 0.3, 0.2)$$

$$\beta_2 = (0.4, 0.3, 0.2, 0.1)$$
$$\beta_3 = (0.7, 0.1, 0.1, 0.1).$$

Let the position vector corresponding to the matrix $\Psi$ be $\beta = (0.6, 0.2, 0.1, 0.1)$; according to the FR 1-norm formula, we have

$$\begin{aligned} N(\beta_1, \beta) &= 1 - \frac{1}{4}\sum_{i=1}^{4} |\beta_1(u_1) - \beta(u_1)| \\ &= 1 - \frac{1}{4}(|0.2 - 0.6| + |0.3 - 0.2| + |0.3 - 0.1| + |0.2 - 0.1|) = 0.80, \end{aligned}$$

where $u_1 \in U$. Likewise, we have $N(\beta_2, \beta) = 0.90$, $N(\beta_3, \beta) = 0.95$.

According to the selected-approach principle, the vector $\beta$ is very close to $\beta_3$, and then $\beta$ is controlled by the high-level radars, so it is an optimal control vector for the 'position' of target 2. Similarly, the control vector $\beta$ is also an optimal control vector for target 3 and target 4. Then, the control matrix $\Psi$ is believed to be an optimal control matrix for the targets 2, 3 and 4. Here we regard the matrix $\Psi$ as an optimal control matrix for targets.

So, according to the above calculation of $b_1, b_2, b_3$ and $b_4$ acquired and the genetic algorithm, the overall better control is obtained with

$$b = b_1 + b_3 + b_4. \tag{3}$$

Again, from a decision rule for considering good control, we first obtain the decision rule, and then select the optimal control. The decision rules are given as follows:

The equivalence class of condition attribute $C_a$ and decision attribute $D_a$ are obtained respectively as $U/C_a = \{X_1, X_2, X_3, X_4\}$, where $X_1 = \{1\}$, $X_2 = \{2\}$, $X_3 = \{3\}$, $X_4 = \{4\}$, $U/D_a = \{Y_1, Y_2, Y_3\}$, where $Y_1 = \{1\}$, $Y_2 = \{2\}$, $Y_3 = \{3, 4\}$.

Then the decision rule is:

$r_{11}$: (1, Position, 0.5) $\bigwedge$ (1, Speed, 0.3) $\bigwedge$ (1, Acceleration, 0.2) $\to$ (1, Very good), i.e., IF the degrees of membership of the position and speed are greater than that of the acceleration for target 1, THEN the control of radars for target 1 is very good, and the decision degree is $\rho(X_1, Y_1) = |Y_1 \bigcap X_1|/|X_1| = 1$; then $r_{11}$ is a certain decision rule.

Similarly, we have $r_{22}$, $r_{33}$ and $r_{43}$, and the corresponding decision degrees are $\rho(X_2, Y_2) = 1$, $\rho(X_3, Y_3) = 1$ and $\rho(X_4, Y_3) = 1$, respectively; then $r_{22}$, $r_{33}$ and $r_{43}$ all are certain decision rules.

The combination rule for radars controlling the targets is:

Let 'the control of radars is better' act as a criterion for whether we select a radar or a group of radars. According to the genetic algorithm [1], the 'better' and 'very good' radars are used in combination in the genetic algorithm, or the decision rule that is better or very good is selected for the control of targets. Thus, a fine radar association is acquired so as to accomplish better control of the targets.

According to the above rule, the decision rules $r_{11}$, $r_{33}$ and $r_{43}$ are selected, that is to say, the above equality (3) is obtained. If the motion state of the target is given by the vector $X$, based on the above FR control rule, then the state of the target is controlled by $X_t = b \circ X$ at time $t$.

## 4. PS control algorithm

### 4.1. Systems model

Here, the systems model consists of $r$ regular models, i.e., $M = \{m^{(1)}, m^{(2)}, \ldots, m^{(r)}\}$. There are $m$ measurements at time $k$, i.e., $Z^k = \{z_1, z_2, \ldots, z_m\}$. $m_k = m^{(i)}$ is a controller model that is working constantly at time $k$.

The state equation of the $i$th model of a dynamical system can be described as

$$x_i(k+1) = F_i(k)x_i(k) + G_i(k)w_i(k), \quad \forall m_i \in M. \tag{4}$$

The measurement equation is

$$z_i(k) = H_i(k)x_i(k) + \nu_i(k), \quad \forall m_i \in M. \tag{5}$$

Here $m_i$ is a current model that is working, $x_i(k) \in R^n$ is an $n \times 1$ state vector; $F_i(k)$ is an $n \times n$ state transition matrix, $G_i(k)$ is an $n \times p$ process noise sequence transition matrix, $w_i(k)$ is a $p \times 1$ process noise, $z_i(k) \in R^m$ is an $m \times 1$ measurement vector, $H_i(k)$ is an $m \times n$ measurement matrix to link output, $\nu_i(k)$ is an $m \times 1$ measurement noise, $w_i(k)$ and $\nu_i(k)$ all are to have a zero mean and positive covariance matrix, and the covariance matrices are $W_i(k)$ and $V_i(k)$ with a Gaussian process noises and measurement noises, respectively, i.e.,

$$E[w_i(k)] = 0 \quad \text{and} \quad E[w_i(k)w_i'(l)] = W_i(k)\delta_{kl},$$
$$E[\nu_i(k)] = 0 \quad \text{and} \quad E[\nu_i(k)\nu_i'(l)] = V_i(k)\delta_{kl}.$$

Here

$$\delta_{kl} = \begin{cases} 1, & k = l \\ 0, & k \neq l. \end{cases}$$

### 4.2. Step of PS algorithm

The PS algorithm consists of $r$ target models and $r$ controller models. Every controller model corresponds to a different target model. The algorithm consists of four parts:

(1) Model-conditioned reinitialization:

According to the predicted value of the former recurrent state of each controller and its covariance matrix and the probability of the controller model of the current state, the initial state and initial covariance matrix of the current controller are calculated. Then we initialize its state and covariance again. Some relevant deriving equations for the $i$th model are denoted as follows:

The predicted probability of the model is $\mu_{k|k-1}^{(i)} \triangleq \Pr(m_k^{(i)}|z^{k-1}) = \sum_j \pi_{ji}\mu_{k-1}^{(j)}$, i.e., the $\mu_{k|k-1}^{(i)}$ that is the probability of the $i$th model at current time $k$ is predicted from the measurements from time 1 to time $k-1$, where $\mu_{k-1}^{(j)} = \Pr(m_{k-1}^{(j)}|z^{k-1})$ is the probability of the model at time $k-1$, $\mu_0^{(j)} = \Pr(m_0 = m^{(j)})$ is known, $\pi_{ji} = \Pr(m_k = m^{(i)}|m_{k-1} = m^{(j)})$ is the probability of state transition of the model from the model $m_{k-1}$ at time $k-1$ to the model $m_k$ at time $k$ and it depends on the model $m_k$, i.e., it is a probability of transition between models, regarded as a known condition.

The mixing weight $\mu_{k-1}^{j|i}$ of the controller model is calculated using

$$\mu_{k-1}^{j|i} \triangleq \Pr(m_{k-1}^{(j)}|m_k^{(i)}, z^{k-1}) = \frac{\Pr(m_k^{(i)}|m_{k-1}^{(j)})\Pr(m_{k-1}^{(j)}|z^{k-1})}{\Pr(m_k^{(i)}|z^{k-1})} = \frac{\pi_{ji}\mu_{k-1}^{(j)}}{\mu_{k|k-1}^{(i)}}.$$

On the basis of the measurements of all $k-1$ historical times, if the controller model is estimated to be the model $m_k = m^{(i)}$ at next time $k$, then the probability of the model $m_{k-1} = m^{(j)}$ at current time $k-1$ is $\mu_{k-1}^{j|i}$.

The one step predicted equation of state estimate for the target model is

$$\overset{\Lambda}{x}{}^{(i)}_{k+1|k} = \overset{\Lambda}{x}{}^{(i)}(k+1|k) = F_i(k)\overset{\Lambda}{x}{}^{(i)}(k|k).$$

The one step predicted error equation of state for the target model is

$$\widetilde{x}^{(i)}(k+1|k) = x_i(k+1) - \overset{\Lambda}{x}{}^{(i)}(k+1|k) = F_i(k)\widetilde{x}^{(i)}(k|k) + G_i(k)w_i(k).$$

The step predicted covariance of error for the target model is

$$\begin{aligned} P_{k+1|k}^{(i)} &= E[\widetilde{x}^{(i)}(k+1|k)\widetilde{x}^{(i)\prime}(k+1|k)|z^k] \\ &= F_i(k)P_{k|k}^{(i)}F_i'(k) + G_i(k)W_i(k)G_i'(k), \end{aligned}$$

where $P_{k|k}^{(i)} = E[\widetilde{x}^{(i)}(k|k)\widetilde{x}^{(i)\prime}(k|k)|z^k]$.

The mixing estimate for the target model is

$$\begin{aligned}\overline{x}_{k-1|k-1}^{(i)} &\triangleq E[x_i(k-1)|m_k^{(i)}, z^{k-1}] \\ &= \sum_j E[x_i(k-1)|m_{k-1}^{(j)}, z^{k-1}]\,\Pr(m_{k-1}^{(j)}|m_k^{(i)}, z^{k-1}) \\ &= \sum_j \overline{x}_{k-1|k-1}^{(j)} \mu_{k-1}^{j|i}.\end{aligned}$$

The mixing covariance of error for the target model is

$$\begin{aligned}\overline{P}_{k-1|k-1}^{(i)} &\triangleq E\left[\left(x_i(k-1) - \overline{x}_{k-1|k-1}^{(i)}\right)\left(x_i(k-1) - \overline{x}_{k-1|k-1}^{(i)}\right)' |z^{k-1}\right] \\ &= \sum_j \left[P_{k-1|k-1}^{(j)} + E\left(\overline{x}_{k-1|k-1}^{(i)} - \hat{x}_{k-1|k-1}^{(j)}\right)\left(\overline{x}_{k-1|k-1}^{(i)} - \hat{x}_{k-1|k-1}^{(j)}\right)'\right] \mu_{k-1}^{j|i}.\end{aligned}$$

(2) Model-conditioned interaction:

Choosing the corresponding controller [22,23] according to the target model, each target model has a corresponding controller in the algorithm; all controllers run side by side.

The predicted state of the model is calculated using

$$\hat{x}_{k|k-1}^{(i)} = F_{k-1}^{(i)} \overline{x}_{k-1|k-1}^{(i)}, \quad \text{where } F_{k-1}^{(i)} = F_i(k-1).$$

The corresponding predicted covariance of error for the state of the model is calculated using

$$P_{k|k-1}^{(i)} = F_{k-1}^{(i)} \overline{P}_{k-1|k-1}^{(i)} (F_{k-1}^{(i)})' + G_{k-1}^{(i)} W_{k-1}^{(i)} (G_{k-1}^{(i)})', \quad \text{where } G_{k-1}^{(i)} = G_i(k-1).$$

The error of the measurement is

$$\tilde{z}_k^{(i)} = z_i(k) - H_k^{(i)} \hat{x}_{k|k-1}^{(i)}, \quad \text{where } H_k^{(i)} = H_i(k).$$

The corresponding covariance of the measure error is

$$S_k^{(i)} = E[\tilde{z}_k^{(i)} (\tilde{z}_k^{(i)})'] = H_k^{(i)} P_{k|k-1}^{(i)} (H_k^{(i)})' + V_k^{(i)}.$$

So, the control gain $K_k^{(i)}$ is obtained from

$$K_k^{(i)} = P_{k|k-1}^{(i)} (H_k^{(i)})' (S_k^{(i)})^{-1}.$$

The updated state is obtained from

$$\hat{x}_{k|k}^{(i)} \triangleq \hat{x}_{k|k-1}^{(i)} + K_k^{(i)} \tilde{z}_k^{(i)}.$$

The updated covariance is also obtained from

$$P_{k|k}^{(i)} = P_{k|k-1}^{(i)} - K_k^{(i)} S_k^{(i)} (K_k^{(i)})'.$$

(3) Probability update of model:

Since the PS algorithm is based on a recurrent algorithm, it needs to calculate the probability update of the controller. On the basis of the current likelihood function and the former recurrent probability of the controller, we can obtain the new probability of the controller while calculating the present one.

Assume that the likelihood function of controller model is a Gaussian mixing density:

$$L_k^{(i)} \triangleq \Pr[\tilde{z}_k^{(i)} | m_k^{(i)}, z^{k-1}] \overset{\text{assume}}{=} \frac{\mathrm{e}^{-(1/2)(\tilde{z}_k^{(i)})'(S_k^{(i)})^{-1}\tilde{z}_k^{(i)}}}{|2\pi S_k^{(i)}|^{1/2}}.$$

On the basis of Bayes' formula, we have the probability of the controller model

$$\mu_k^{(i)} \triangleq \Pr(m^{(i)}|z^k) = \frac{\mu_{k|k-1}^{(i)} L_k^{(i)}}{\sum\limits_j \mu_{k|k-1}^{(j)} L_k^{(j)}}.$$

(4) Overall output:

The mixing overall estimate and overall covariance matrix are calculated from the state estimate and covariance of the current $r$ target models, and controller models and the current probability of the model that have been obtained. The overall estimate and overall covariance of the model can be treated as the final output with this circulation.

On the basis of the above calculation, the overall output for the overall estimate and overall covariance are obtained as follows, respectively:

The overall estimate for the target model is $\overset{\Lambda}{x}_{k|k} = \sum_i \overset{\Lambda}{x}{}_{k|k}^{(i)} \mu_k^{(i)}$.

The corresponding overall covariance of the error for the target model is

$$P_{k|k} = E\left[\left(x_i(k) - \overset{\Lambda}{x}_{k|k}\right)\left(x_i(k) - \overset{\Lambda}{x}_{k|k}\right)' |z^k\right] = \sum_i \left[P_{k|k}^{(i)} + E\left(\overset{\Lambda}{x}_{k|k} - \overset{\Lambda}{x}{}_{k|k}^{(i)}\right)\left(\overset{\Lambda}{x}_{k|k} - \overset{\Lambda}{x}{}_{k|k}^{(i)}\right)'\right]\mu_k^{(i)}.$$

The inverse of the covariance matrix is called the information matrix.

Assume

$$P_{i,k|k} = \left[P_{k|k}^{(i)} + E\left(\overset{\Lambda}{x}_{k|k} - \overset{\Lambda}{x}{}_{k|k}^{(i)}\right)\left(\overset{\Lambda}{x}_{k|k} - \overset{\Lambda}{x}{}_{k|k}^{(i)}\right)'\right]\mu_k^{(i)}, \quad \overset{\Lambda}{x}_{i,k|k} = \overset{\Lambda}{x}{}_{k|k}^{(i)} \mu_k^{(i)}.$$

The state of the target is controlled by the information matrix. The control is described by the following formula:

$$P_{k|k}^{-1} \cdot \overset{\Lambda}{x}_{k|k} = \sum_i P_{i,k|k}^{-1} \cdot \overset{\Lambda}{x}_{i,k|k}. \tag{6}$$

By the above formula (6), the bigger $P_{i,k|k}$ is, the smaller $P_{i,k|k}^{-1}$ is; thus, the $P_{i,k|k}$ operation is less effective for target control.

From the above discussion (and Refs. [9,11,14,17,20,22–26,29] for more detailed information), according to prior knowledge, the fuzzy rough theory and the probability theory are used in different levels in application — that is, when all probabilities of conditional items and statistical distributions are known, the probability theory is usually used; however, the fuzzy theory, the rough sets theory and the fuzzy rough theory are separately utilized when some probabilities of conditional items and statistical distributions are unknown. In addition, the fuzzy rough theory and the probability theory can be used simultaneously when the prior knowledge satisfies the requirement of probability.

## 5. Simulations and performance analysis for the control algorithms

### 5.1. Simulation

#### 5.1.1. Target model establishing

On the basis of the target tracking, the target control can be completed. In order to simplify the simulation, here, three target models and three radars are chosen. According to the above Sections 3 and 4, the target control is obtained by target tracking, and the target tracking is simulated as follows.

The state vector for the target models has two dimensions: position and speed, i.e., $X = \begin{pmatrix} x & \dot{x} & y & \dot{y} \end{pmatrix}'$. The measurement vector is $Z = \begin{pmatrix} x & y \end{pmatrix}'$.

According to [22,23], here, the state transition matrix is chosen as follows:

$$F_1 = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad F_2 = \begin{pmatrix} 1 & \dfrac{\sin\omega_2 T}{\omega_2} & 0 & \dfrac{1-\cos\omega_2 T}{-\omega_2} \\ 0 & \cos\omega_2 T & 0 & \sin\omega_2 T \\ 0 & \dfrac{1-\cos\omega_2 T}{\omega_2} & 1 & \dfrac{\sin\omega_2 T}{\omega_2} \\ 0 & -\sin\omega_2 T & 0 & \cos\omega_2 T \end{pmatrix},$$

$$F_3 = \begin{pmatrix} 1 & \dfrac{\sin\omega_3 T}{\omega_3} & 0 & \dfrac{1-\cos\omega_3 T}{-\omega_3} \\ 0 & \cos\omega_3 T & 0 & \sin\omega_3 T \\ 0 & \dfrac{1-\cos\omega_3 T}{\omega_3} & 1 & \dfrac{\sin\omega_3 T}{\omega_3} \\ 0 & -\sin\omega_3 T & 0 & \cos\omega_3 T \end{pmatrix}.$$

The three models have the same process noise transition matrix $G$, measurement matrix $H$, process noise covariance $W$ and measurement noise covariance $V$. These matrices are denoted as follows, respectively:

$$G = \begin{pmatrix} \dfrac{T^2}{2} & 0 \\ T & 0 \\ 0 & \dfrac{T^2}{2} \\ 0 & T \end{pmatrix}, \qquad H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$W = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}, \qquad V = \begin{pmatrix} 3\times 10^3 & 0 \\ 0 & 3\times 10^3 \end{pmatrix}.$$

*5.1.2. Initialization conditions*

Initialize the state vector $x_0$, covariance matrix $P_0$ and transition probability matrix $\pi_{ij}$ of the model and probability $\mu_0$ of the model as follows, respectively:

$$x_0 = \begin{pmatrix} 10^3 & 100 & 10^3 & 100 \end{pmatrix}', \qquad \mu_0 = \begin{pmatrix} 0.5 & 0.3 & 0.2 \end{pmatrix}',$$

$$P_0 = \begin{pmatrix} 10^3 & 0 & 0 & 0 \\ 0 & 600 & 0 & 0 \\ 0 & 0 & 10^3 & 0 \\ 0 & 0 & 0 & 600 \end{pmatrix}, \qquad \pi_{ij} = \begin{pmatrix} 0.9 & 0.05 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.05 & 0.15 & 0.8 \end{pmatrix}.$$

*5.1.3. Motion state and simulation*

Let the targets undergo rectilinear motion at a uniform velocity and take a turn to the right or the left in the motion at a uniform velocity, where the deviation turning rates are $\omega_2 = 5^\circ$ and $\omega_3 = -5^\circ$.

In the simulation of the FR algorithm, the control weights of the radars 1, 2 and 3 for the target are $a_1 = (0.5, 0.3, 0.2)$, $a_2 = (0.1, 0.3, 0.6)$ and $a_3 = (0.1, 0.5, 0.4)$, respectively. The true system control matrix is

$$\Psi = \begin{pmatrix} 1-e^{-\omega_2 t} & 1-e^{-\omega_2(t+1)} & \cdots \\ e^{-\omega_2 t} & e^{-\omega_2(t+1)} & \cdots \\ e^{\omega_3 t} & e^{\omega_3(t+1)} & \cdots \end{pmatrix},$$

where $\omega_2 = 5^\circ$ and $\omega_3 = -5^\circ$, $t$ is a time variable, $\Psi$ is a $3\times T_0$ matrix and $T_0$ is the simulation sampling, that is 120 times; the sampling rate $T$ is 2 s. The simulation results are shown in Table 2, Figs. 1 and 2.

*5.2. Simulation results and analysis*

The tracking curves of the FR and PS algorithms are shown in Fig. 1. From Fig. 1, the PS algorithm is better than the FR algorithm on multiple-target control. The tracking curve of the PS algorithm is basically the same as the true

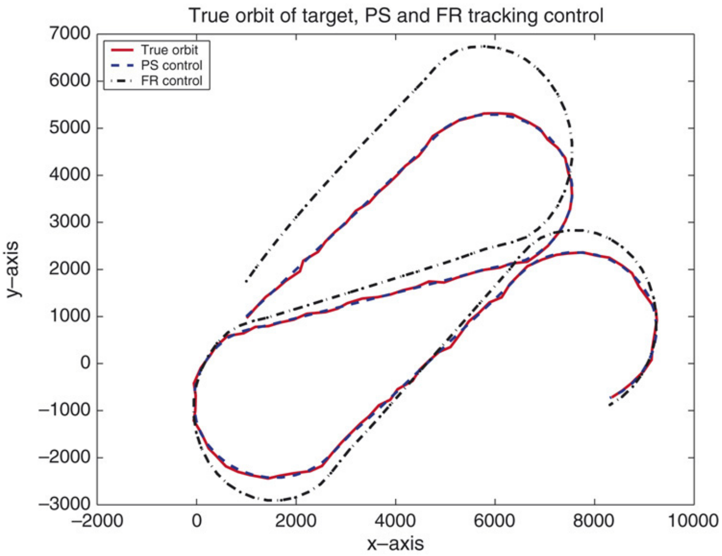


Fig. 1. True orbit of target; PS and FR tracking control.

curve of the target orbit, while the FR algorithm is not so good for target tracking. The error curves of the PS and FR algorithms are shown in Fig. 2; from Fig. 2, we see that the error of the FR algorithm is bigger than that of the PS algorithm.

Using the simulation results, the FR algorithm is compared with the PS algorithm for target control; its greatest advantage is that it not only has faster processing speed, and lower memory capacity and communications traffic, but also is applicable to a system with larger navigation, calibration, conversion and transmission error. However, the biggest disadvantage of the FR algorithm is that the setting up of system parameters is complicated. For example, it needs a great deal of simulations to determine some parameters of the membership function and control matrix. At the same time, its control effects are not as good as those of the PS algorithm according to the target tracking. The greatest advantage of the PS algorithm is that it is able to integrate new knowledge and old knowledge, but its biggest disadvantage is that we must know the exact probability or prior probability of conditional items and statistical distributions.

On the basis of the above analysis, the FR algorithm is an optimal choice for target control when some probabilities of conditional items and statistical distributions are unknown or the requirement of precision of control is not too high, because it has a lot of advantages as compared with the probabilistic algorithm; in addition, some probabilities of conditional items and statistical distributions are usually obtained with difficulty.

### 5.3. *Comprehensive comparison*

To evaluate the overall performance of two algorithms, we adopt a combination of quantitative analysis and qualitative analysis methods, and synthesize a comparison based on the computing speed, memory capacity and communications traffic, tracking effect and parameter setup requirements. We evaluate the advantages and disadvantages of the PS algorithm and FR algorithm for control. Table 2 gives the results of the comprehensive comparison.

In Table 2, $\overline{e}$ is a mean absolute value of the tracking error. The computing speed denotes the computing time, and the computing time is obtained only by the calculation of each algorithm. In simulation, the computer that is used is the Pentium 4 with 512M memory, and the programming language which is used is MATLAB. The memory capacity and communications traffic are estimated approximately, based on the process of computing and complexity of every algorithm respectively. From the results of Table 2 it is seen that the memory capacity and communications traffic are closely related. The tracking effects are shown by the average of the absolute value of the tracking error for every algorithm in simulation. The parameter setup requirements imply that some parameters that an algorithm includes need to be set on the basis of the actual environment.

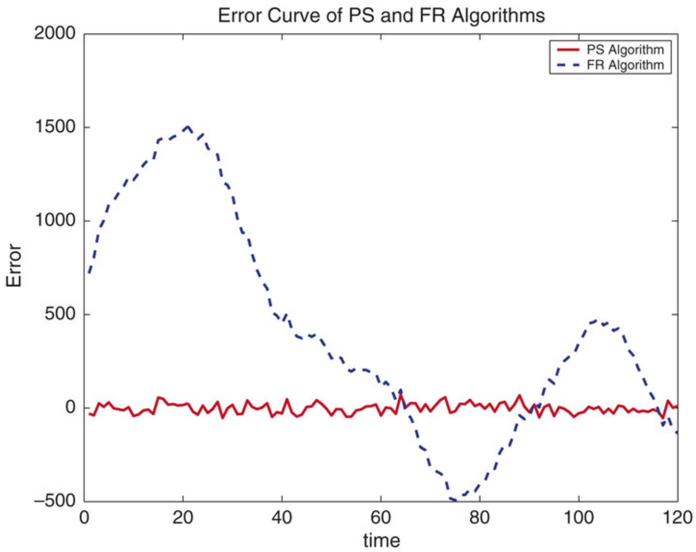


Fig. 2. Error curve of PS and FR algorithms.

Table 2
Comprehensive comparison of PS and FR algorithms on control

| Algorithm | Speed | Memory | Traffic | Tracking effect ($\bar{e}$) | Parameters setup |
|---|---|---|---|---|---|
| PS | 0.0156 | Middle | Middle | 27.8184 | No |
| FR | 0.0006 | Low | Low | 555.5978 | Yes |

## 6. Conclusions

In target control, there are various algorithms; what algorithm we should choose in different applications is an unsolved problem. This paper studies an aspect of the problem, i.e., discusses a new control algorithm from the combination of fuzzy theory and rough sets theory, and gives a comparison of the new control algorithm and the probability algorithm. Here, the new control algorithm is called the FR algorithm. What the FR algorithm on target control does is first select several optimum radars according to the maximum-membership principle, and then finish by controlling the target using the fusion of information based on a genetic algorithm. But the PS algorithm utilizes the characteristics of mean and covariance in order to finish the target control using accurate numerical computations. According to the state equation and measurement equation of the target model, and by providing the initial value, sample, probability of the model, transition probability, total-probability formula, total-expectation formulae, fusion algorithm, etc., the PS algorithm carries out the calculation.

According to the simulation results, comparing the FR and PS algorithms for target control, the algorithm based on fuzzy mathematics has the faster processing speed, and lower memory capacity and communications traffic, but its control effects are not as good as those of the PS algorithm.

The FR approach in handling some problems is flexible, but it lacks solid, systematic weight update, while the PS approach has a built-in solid mechanism for sequential update of the weights thanks to Bayes' rule. For some targets that are uncertain, vague, cannot be computed on the basis of prior knowledge but can be inferred from some logic rules and expert systems, we make use of the FR approach. However, for some targets that are uncertain, probabilistic, and can be computed using definite mathematical formulas according to prior knowledge, we make use of the PS approach. Thus, we not only understand their difference from the theoretical angle, but also know what kind of field they are suitable for application in.

The theories of FS and RS all are novel, effective, soft scientific approaches; then the theory of FRS is a more unique and effective soft scientific approach, and it can analyze and deal with some fuzzy and incomplete information. The processing technology of FRS as regards fuzzy uncertain information, combination of it and other soft science and soft computing approaches, and its combination with PS has enormous potentialities for application. Because the investigations of theories and applications on FRS are still at a preliminary developing stage now, the theories and applications of the combination of FRS and PS are in an embryonic state; some technological questions, such as those of the dispersed normalization of attribute values of the continuous data, choosing optimum discontinuous points, knowledge acquisition, etc., will require a lot of scholars to further probe into the combination of the two and carry out further diversified investigations in the future. These problems to be solved will have direct impact on the development of FRS and probability theory, and will show the further and great prospects for wide applications.

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